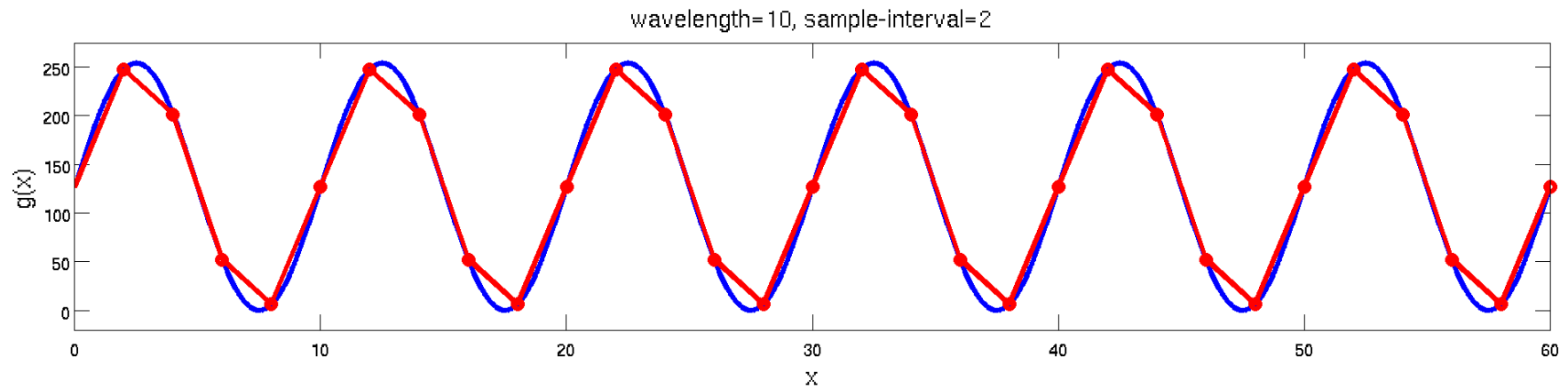


Slide 8

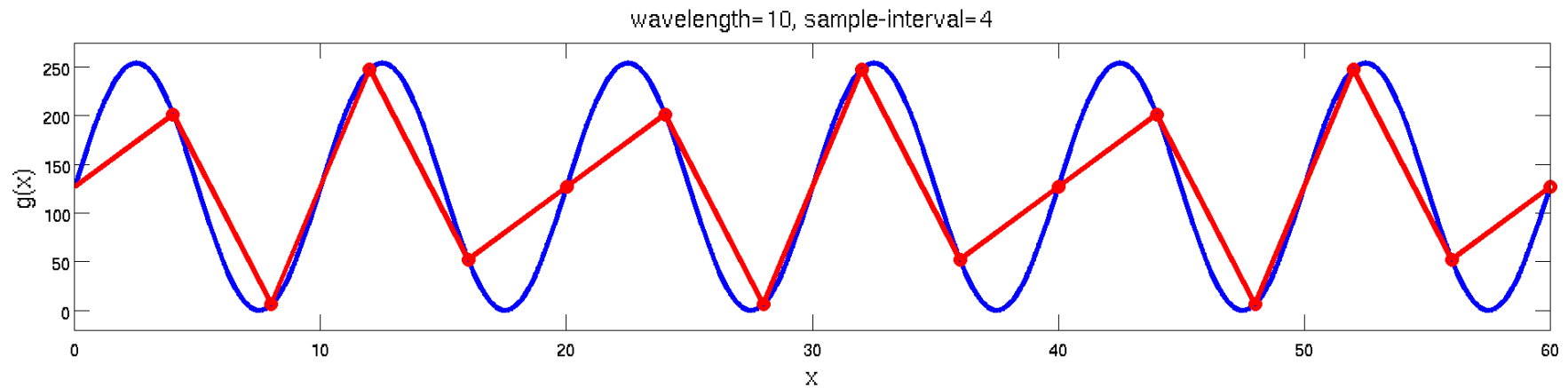
Sampling: Moiré Patterns cont.

- 1D-example of Moiré patterns:



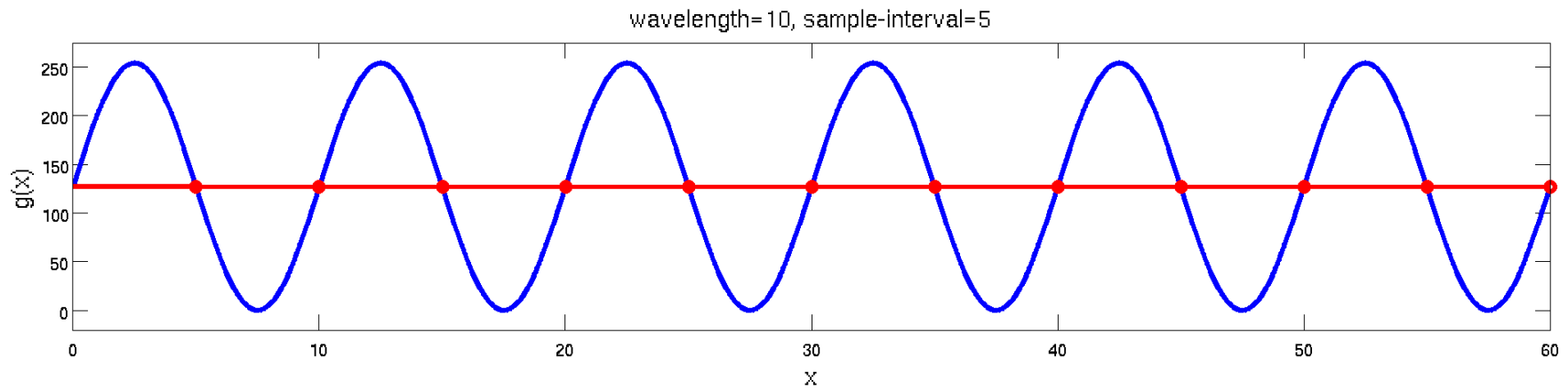
Sampling: Moiré Patterns cont.

- 1D-example of Moiré patterns:



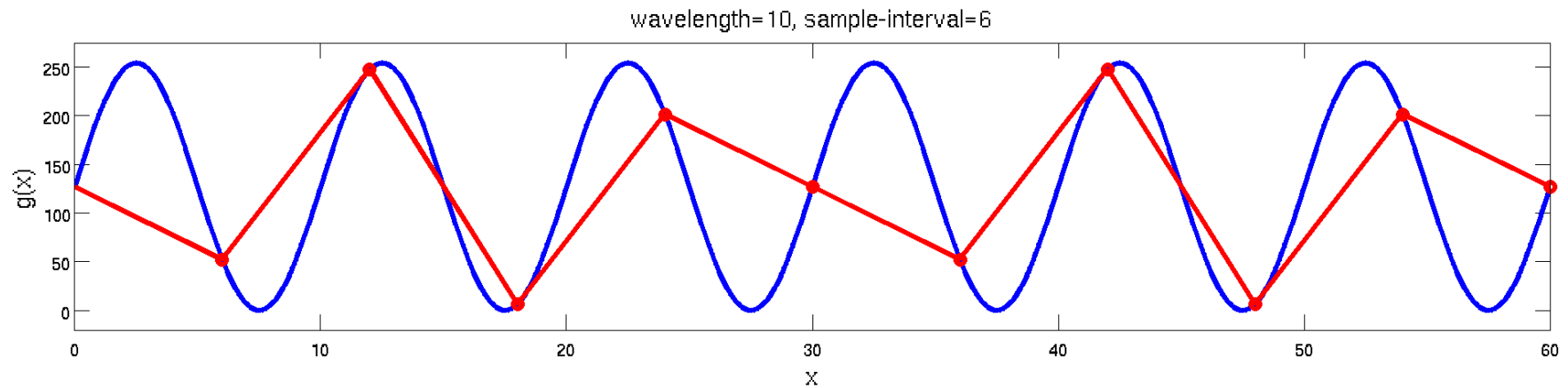
Sampling: Moiré Patterns cont.

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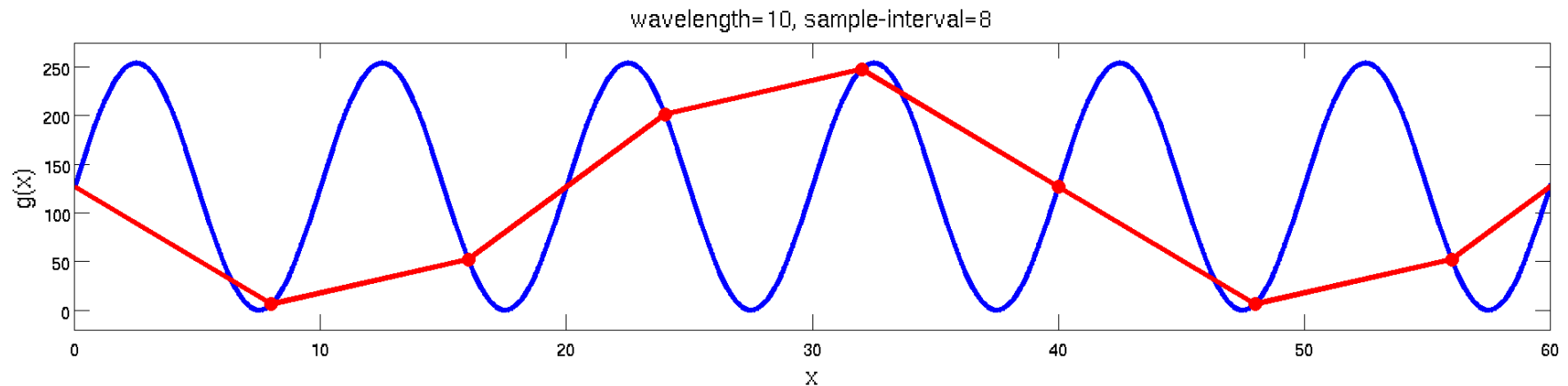
Sampling: Moiré Patterns cont.

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Sampling: Moiré Patterns cont.

- 1D-example of Moiré patterns:



The occurrence of Moiré patterns depends on the sampling rate compared to the maximal frequency of the signal (image)

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Convolution of Images

- Example

 $g :$

3	4	2	2	2	2	4
5	2	1	1	1	0	2
6	3	2	2	0	1	3
4	4	8	5	4	7	3
6	6	7	8	9	9	7

 $f :$

2	2	1
1	0	0
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 $g * f :$

	30	26	15	12	18	
	47	39	30	32	30	
	61	52	53	57	45	

$$(g * f)(5, 1) =$$

$$+ f(-1, -1)g(6, 2) + f(0, -1)g(5, 2) + f(1, -1)g(4, 2)$$

$$+ f(-1, 0)g(6, 1) + f(0, 0)g(5, 1) + f(1, 0)g(4, 1)$$

$$+ f(-1, 1)g(6, 0) + f(0, 1)g(5, 0) + f(1, 1)g(4, 0)$$

$$(g * f)(u, v) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} f(k, l)g(u - k, v - l)$$

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Convolution of Images

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Convolution of Images

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Convolution of Images

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Convolution of Images

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boundary pixels are typically left free since convolution requires evaluation of pixels outside of image g

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